

# MCG Newsletter #3

October 2012

Newsletter of the International Group for Mathematical Creativity and Giftedness

## *Newsletter* of the International Group for Mathematical Creativity and Giftedness



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# **News from MCG-7 Conference**

# Busan, Korea, July 15-18



#### **Emilia Velikova**

MCG Secretary

The program of the conference, the names of participants and the presented papers can be found at the conference website: <u>http://www.mcg7.org</u>

**Special thanks to Prof. Choi Jong Sool** for the excellent organization of the conference and warm hosting of the conference delegates.

The results of the elections that took place during at the general meeting of MCG are the following

Position	Name	Country	Years of duty
Officers			
President of MCG	Roza Leikin	Israel	2012-2016
Vice President MCG	Linda Sheffield	USA	2010-2014
Secretary	Emiliya Velikova	Bulgaria	2010-2014
Treasurer	Peter Taylor	Australia	2012-2016
International committee mer	nbers		
	Francisco Bellot-Rosado	Spain	2010-2014
	Dace Bonka	Lithuania	2012-2016
	Romualdas Kasuba	Lithuania	2010-2014
	Boris Koichu	Israel	2010-2014
	Vincent Matsko	USA	2010-2014
	Hartwig Meissner	Germany	2012-2016
	Marianne Nolte	Germany	2012-2015
	Demetra Pitta-Pantazi	Cyprus	2010-2014
	Choi Jong Sool	South Korea	2010-2014
	Mark Saul	USA	2012-2014
	Mihaela Singer	Romania	2012-2016
	Kim Seong-A	South Korea	2012-2016

The new MCG Board would like to thank **Prof. Hartwig Meissner – the Establishing MCG President –** for his great contribution to the organization of the Group and promotion of the MCG field.



# **Important Upcoming Events**

# MCG 8

The 8th International Conference on Mathematical Creativity and Giftedness The University of Denver Campus in Denver, Colorado July 28-30, 2014.

NAGC 2012 http://www.nagc.org

The 2012 Annual Convention of the National Association for Gifted Children (USA)

Denver, Colorado November 15 - 18, 2012

# CERME 8 http://www.cerme8.metu.edu.tr/

Eighth Congress of European Research in Mathematics Education <u>WG 8 – Mathematical potential, creativity and talent</u>

Starlight Convention Center, Thalasso & Spa Hotel in Manavgat-Side, Antalya, Turkey February, 6-10, 2013.

# NCTM 2013 www.nctm.org

2013 Annual Meeting & Exposition of the National Council of Teachers of Mathematics (USA)

> Denver, Colorado April 17-20, 2013



# **New MCG Activities**

Roza Leikin, MCG President

## Linda Sheffield, MCG Vice president

We are happy to introduce activities and structures that new IC of MCG establishes in order to promote exploring and nurturing mathematical creativity and giftedness all over the world. Among new strictures and activities are

- THE TREASURER TEAM, *The Treasurer* Peter Taylor
- AYR-MCG WT: Working Team for the Assistance to Young Researchers in Mathematical Creativity and Giftedness WG Leaders: Boris Koichu, Marianne Nolte
- STIN-MCG WT: Working Team for Supporting Teachers in Nurturing Mathematical Creativity and Giftedness

WG Leaders: Mark Saul, Mark Applebaum, Choi Jong Sool

- WEBSITE-MCG TEAM, Website Managers: Mark Applebaum, Viktor Freiman
- INTERNATIONAL SUMMER CAMPS, Coordinator Florence Mihaela Singer

## THE TREASURER TEAM

#### Peter Taylor - the Treasurer- MCG

The Treasurer's Team will soon commence a discussion to explore the following areas

- \* whether the charging of membership fees would be an enhancement for MCG
- \* whether the current constitutional model is the most appropriate for MCG or whether some more formal model, maybe even corporatization, might be beneficial
- \* any related matter raised by the membership.

The Team currently comprises Peter Taylor, Dace Bonka, Choi Jong Sool and Roza Leikin. I note that as a result of the recent conference in Busan running at a surplus the MCG has received this surplus as a generous donation. This money is being held in trust by the Australian Mathematics Trust, which is prepared to mind it and add interest at the rate it receives from the bank, until called by MCG. So members may wish to express a view about this also.

Please make any submission to the Treasurer's Team to Peter Taylor at pjt013@gmail.com.



## AYR-MCG WG

Working Team for the Assistance to Young Researchers in Mathematical Creativity and Giftedness

# WG Leaders: Boris Koichu, Technion – Israel Institute of Technology

Marianne Nolte, Uniz

University of Hamburg, Germany

This announcement is addressed to either young or experienced researchers in the field of mathematical creativity and giftedness, as well as to those who perceive themselves as both young *and* experienced researchers, as the authors of this announcement do.

#### To a young researcher

*Imagine that you decided to submit a research paper* to a highly reputed international journal in the field of your study. Imagine that four or five months later the reviewing set – a decision letter from the editor and three reviews by anonymous reviewers – appear in your mail box. You open the set with excitement, digest the decision, and then find in the reviews some of the following:

The paper is about an interesting and important topic... However, the paper in its present form raises serious concerns related to its theoretical background and contribution.

Or:

The second research question is particularly important... Unfortunately, it is out of reach of the implemented research methodology.

Or:

This reviewer was impressed by the clarity of the introduction and by the detailed procedure the author developed for analyzing his or her data. I am much less satisfied with presentation and discussion of the results of the study and suggest major restructuring of these sections.

Now imagine that some of the reviewers' criticism could be anticipated and that you are given a chance to improve your paper prior to formally submitting it to a journal.

Sounds great, isn't it?

#### To an experienced researcher

*Imagine that you are a young researcher*...Your chance to be advised and assisted at the early stages of your career depends on the good will of your experienced colleagues.

Many of us gratefully remember a piece of advice from a caring stranger, in addition to the immense advice that came from our research advisors. Our working team needs you in the role of such a caring stranger for young MCG researchers.

#### To the MCG scholarship

The main goal of the AYR-MCG working team is to assist young researchers in conducting their first studies and preparing their first research publications. Our additional goal is to facilitate the development of the community of practice involving MCG researchers from different countries and at different stages of their careers. We hope that the team will accumulate the expertise of the members of the MCG International Group in conducting and publishing high-quality research. The work of the team will be organized in two directions.



#### 1. Assist and be assisted

We will respond – within reasonable time and at reasonable level of detail – to requests regarding the informal evaluation of research reports and proposals prior to their submission. Young researchers may also seek in the AYR-MCG team advice regarding the journals for publishing or conferences for presenting their results, methodological issues, etc. The requests will be addressed by us, the authors of this announcement, or will be redirected to other researchers cooperating with the AYR-MCG team. Currently, we are able to respond to requests written in English, German, Russian and Hebrew.

We believe in expert advice, and we also believe in sharing experiences within the communities of practice. As it is already done in some well-established research organizations in the field of mathematics education (e.g., MAVI and CERME) we plan to facilitate mutual reviewing processes: once assisted, you may expect an invitation to become an assistant. (Needless to say, you may expect an invitation to assist also if you have not yet been assisted.)

#### 2. Research tips

We plan to occasionally publish on the MCG Website tips on preparing the reports, comments on research methodology, reflections of reviewing processes etc.

If you wish to take part in the work of AYR-MCG WG, that is, to assist or be assisted or share with the MCG members tips based on your research experience, please contact Boris Koichu (bkoichu@technion.ac.il) and Marianne Nolte (Marianne.Nolte@uni-hamburg.de)

#### **STIN-MCG WG**

## Working Team got Supporting Teachers in Nurturing Mathematical Creativity and Giftedness

#### WG Leaders: Mark Saul, Center for Mathematical Talent, Courant Institute of Mathematical Sciences, NYU

Mark Applebaum, Kaye Academic College of Education, Israel Choi Jong Sool, Kaist Academy, Bussan, South Korea

This announcement is addressed to either young or experienced teachers in the field of mathematical creativity and giftedness

This WG is planning to open several forums for

- (a) Promoting teaching for creativity: in many cases, teachers of gifted students, or those with a classroom goal of fostering creativity, work in isolation. This forum will offer a platform from discussion of issues which arise in everyday pedagogical and mathematical life.
- (b) Encouraging teachers who foster creativity: discussing the problems of supporting teachers in this subtle area of teacher education.
- (c) Developing resources for gifted students: gifted students often can not find resources to fit their special needs in their classroom. Meeting with other gifted students and experts in this area can help them. (Note that this is a forum for teachers, not for the students themselves).



(d) Supporting parents of creative or gifted students: A discussion of the need for family support of students who think out of the box. This includes parents of home-schooled students, parents in remote areas, or parents looking for a support network in nurturing their creative or gifted child.

Forums will be intended for brief interchanges among many people. In addition, we plan 'corners', where lengthier essays can be posted, offering deeper insight into experiences and issues:

- ✓ Corner for specialists' (including teachers') essays, sharing practical ideas and experiences
- ✓ Corner for mathematical problems and other resources, written from a pedagogical viewpoint.

If you wish to take part in the work of STIN-MCG WG, that is, to assist or be assisted or share with the MCG members tips based on your teaching experience, please contact Mark Saul (<u>marksaul@earthlink.net</u>), Mark Applebaum (<u>mark@kaye.ac.il</u>), Choi Jong Sool (choijon@kaist.ac.kr)

#### WEBSITE-MCG TEAM

#### Website Managers: Viktor Freiman, University of Moncton, Canada

#### Mark Applebaum, Kaye Academic College of Education, Israel

This working team will take leadership on the development of the MCG website and encouraging broad educational community to share their ideas and experiences in the field of MCG. This WG will provide support to AYR-MCG WG and STIN-MCG WG as well as to other initiatives.

The MCG members are invited to share their ideas about development and promotion of the site by writing an e-mail to Mark (<u>mark@kaye.ac.il</u>) and Viktor (<u>viktor.freiman@umoncton.ca</u>).

#### **ISCIL WT - INTERNATIONAL SUMMER CAMPS AND INFORMAL LEARNING**

#### **Coordinator:** Florence Mihaela Singer, University of Ploiesti, Romania

The ISCIL Working Team of the igMCG is planning to support members of educational community in organization of mathematical competitions, summer camps and other out-of school activities directed at promotion of MCG.

As a start for the work of WT we invite teachers, teacher educators and researchers to attend the 2013 edition of the International Summer Camp *Kangaroo* in Romania.

The International Camp Kangaroo emerged in 1997 to award the winners of the International Competition of Applied Mathematics Kangaroo. Within the expansion of the Kangaroo competition in Romania (and elsewhere), the summer camp has become more and more developed. The camp is organized by the Institute for the Development of Educational Assessment (IDEA) and the Foundation for European Integration Sigma (FIES). The camp becomes a place of meetings between students, teachers, and researchers, where ideas



develop and they are put into practice, with important educational benefits. It involves a high degree of flexibility in order to create an environment for highly effective informal learning.

For more details about Kangaroo competitions and to join ISCIL WT, please contact Florence Mihaela Singer (<u>mikisinger@gmail.com</u>)

# **ELECTRONIC REGISTRATION FORM**

Membership on-line registration form is now available on the MCG website. Please join MCG International group by clicking on membership button at <u>http://igmcg.org/</u> and completing the form.



# News from the12th International Congress on Mathematical Education

# ICME-12 - Topic Study Group 3 - Activities and Programs for Gifted Students

Peter Taylor (Australia) and Roza Leikin (Israel) Chairs TSG3

## Introduction: the aim and the focal topics

The aim of TSG-3 at ICME-12 was to gather educational researchers, research mathematicians, mathematics teachers, teacher educators, designers and other congress participants for the international exchange of ideas related to identifying and nourishing mathematically gifted students. The focal topics presented at the TSG-3 included but were not restricted to theoretical models of giftedness, the relationship between creativity and giftedness and the empirical research that will contribute to the development of our understanding in the field. Participants discussed effective research methodologies and research innovations (e.g., brain research) in the field of mathematical giftedness; the findings of qualitative and quantitative studies related to high mathematical promise, its realization, and the relationship between mathematical creativity and mathematical talent. Additional attention was given to the profiles of the gifted child: their range of interests, ambitions and motivations, social behaviour, how and at what age their giftedness is discovered or developed.

Educators who participated in TSG-3 discussed instructional design directed at teaching the gifted as well as development of appropriate didactical principles. The discussions were focused on the ways that lead students to discover and realize their mathematical talents, and the ways of developing mathematical innovation at high level. The participants discussed mathematical activities that are challenging, free of routine, inquiry-based, and rich in authentic mathematical problem solving; types of mathematics suitable for challenging gifted students; creation of mathematics challenges; out-of-school ways of fostering giftedness, e.g., mathematics clubs, mathematical shows and competitions.

Last but not least we paid attention to teacher education aimed at mathematics teaching that encourages mathematical promise and promotes mathematical talents, including issues of the psychology of teaching talented students, socio-cultural and affective characteristics of the mathematically gifted, and the types of mathematics and pedagogy suitable for educating teachers of gifted.

Participants took part in four sessions. Three sessions (1, 2, and 4) were devoted to research and project presentations and the discussions based on these presentations. Session 3 was organised with round table presentations. In what follows we present main topics of the sessions and some examples of the studies and projects presented at the TSG-3 at ICME-12.



# **Examples and main insights**

#### **Opening the discussion**

Session 1 was devoted to introduction to the central topics of the TSG. Three lectures, by Linda Sheffield, Roza Leikin and Alexander Soifer, opened three main venues of the TSG: international projects for realizations of students' mathematical potential with special emphasis on high mathematical potential (REF), systematic research on characterization of mathematically gifted students, and mathematics for mathematically gifted.

Linda Sheffield's talk "Mathematically Gifted, Talented, or Promising: What Difference Does It Make?" stressed the importance of the developmental perspective of mathematical abilities and the importance of providing each and every student with opportunities to realize these abilities. Based on the position that science, technology, engineering, and mathematics (STEM) are critical to the economy, security, and future of the world, Linda Sheffield argued that we need students who will become adults who understand the complexities of a technological world, who ask the essential questions to safeguard that world, and who will become the leaders, researchers and innovators in the STEM fields of the future. According to Sheffield, too often, in the United States, these students go unrecognized, unmotivated, and under-developed at a time when they are most vital. Sheffield discussed in her presentation whether the way we historically define these future STEM leaders and innovators has an effect upon their growth and development. This talk served as a starting point to the discussion of the international project devoted to the realization of students' intellectual potential related to STEM.

Roza Leikin stressed the importance of conducting systematic and well-designed research on the characteristics of mathematically gifted students. She presented large-scale Multidimensional Examination of Mathematical Giftedness that she conducts with colleagues from the research group in the University of Haifa (Mark Leikin, Ilana Waisman, Shelley Shaul). The presentation was devoted to brain activity (using ERP- Event-Related Potentials - methodology) associated with solving mathematical problems that require transition from a geometrical object to a symbolic representation of its property. Some 43 right-handed male students with varying levels of general giftedness (Gifted-G, Nongifted-NG) and of mathematical expertise (Excelling-E, Non-excelling-NE) took part in the study. The researchers aimed to investigate the differences in brain activation among four groups of participants (G-E, G-NE, NG-E, and NG-NE). The findings demonstrated different patterns of brain activity associated with problem solving among four experimental groups. In educational practice the results suggest that different groups of the study population need specific instructional approaches to realize fully their intellectual potential.

Alexander Soifer claimed that mathematics cannot be taught, it can only we learned by our students while doing it. According to Soifer, the classroom ought to be a laboratory where students actually touch the subject, overcome difficulties, which we sometimes call problem solving. "What kind of problems?" – asked the author, and answered: "here comes Combinatorial Geometry!" It offers an abundance of problems that sound like a "regular" school geometry, but require for their solutions synthesis of ideas from geometry, algebra number theory, and trigonometry and thus they are rich, challenging and insightful, and thus appropriate for the education of mathematically talented individuals.



When the three presenters finished their presentations it became clear that the contrast between the presentations enlightened the importance and openness of the following questions: Who are the mathematically gifted? Can giftedness be developed or rather is it realized? How do different perspectives on giftedness determine research and practice in the education of the mathematically gifted? and What kinds of mathematics problems are most appropriate to mathematically gifted?

#### International experiences and projects for gifted

The second session was devoted to the projects of different kinds directed at educational activities with mathematically advanced students.

Mark Saul described activities of the Center for Mathematical Talent (CMT) at the Courant Institute of Mathematical Sciences (New York University) which was organized in the fall of 2010. Its mandate is to identify and support mathematically talented students in and around the New York City area—especially those from backgrounds where such services have traditionally been weak. The goal at the CMT is to create institutions, materials, and practices that will unlock and nurture these abilities in students, and will have an impact both on their lives as individuals and on the society in which they live.

Ildar Safuyanov reported on the experiences of fostering creativity of pupils in Russia. While the creative approach is understood by the authors and his colleagues as certain abilities and readiness of a person for creating something new, the purpose of educational process at school is the education of a person who would use a creativity approach for solving scientific or practical problems and for thinking independently. According to Safuyanov, differentiated teaching is an effective way of promoting creativity in conditions. Ildar Safuyanov discussed and compared different types of differentiated teaching and provided the audience with examples of internal differentiation by level of mathematical tasks.

Abraham Arcavi presented the Math-by-Mail project which is an online, interactive, extracurricular enrichment program in recreational mathematics conducted by mathematics educators from the Weizmann Institute of Science in Israel (leaders- Yossi Elran, Michal Elran, Naama Bar-On). Participants of the Math-by-Mail project are engaged in a multi-sensory learning experience involving many skills such as comprehension, solving inquiry-based problems and correspondence with mathematicians. The lecture demonstrated the scope of the program, its pedagogical and technological characteristics and its benefits for the talented math student.

Viktor Freiman from the University of Moncton, Canada, shared his innovative experience of designing and conducting professional learning communities with inclusive practices for students who "already know". In his project, mathematically gifted and talented students contribute to the virtual community. Same research findings demonstrated the effectiveness of the suggested approach as well as its complexity.

Duangnamol Tama reported on the project named "The Development and Promotion of Science and Technology Talented Project (DPST)". The project is supported by the the Thailand government. Thus national education focuses its efforts and policies on the national development of science, mathematics, and technology through the promotion of high caliber students in these areas.

At the end of this session the participants were exposed to the variety of approaches and variety of ideas directed at promotion of the mathematically gifted. Further discussion



between the participants of the session was directed at answering the questions: Which features of the programs for mathematically gifted are culturally dependent and which of them are intercultural? Can successful projects from one country be applied in another country with a deferent cultural heritage? Do inclusive programs suit needs of the gifted?

## **Didactical approaches and international perspectives**

At Session 3 participants of the TSG-3 were exposed to different didactical approaches and international perspectives on the education of mathematically advanced students. This was a round tables session. The authors were provided with an opportunity to present their papers several times to different people who were interested in their presentations. The groups changed each 10 minutes and each participant had an opportunity to learn about several works presented at this session. These works included:

- The program of making students create math problems: One of the methods of developing students' abilities to think and express by Nobuo Itoh from Japan,
- The role of student motivation in developing and assessing the acquisition of higher-order thinking skills, by Vincent Matsko, USA
- How the mathematically gifted and talented senior primary school students in Hong Kong understand mathematics? by Wai Lui Ka, Hong Kong
- The research on the mode of motivating the gifted students, by Wang He Nan, , Beijing,
- Enhancing mathematical research in high school, by Laura Morera, Spain
- Mathematical creativity and attachment theory: an interdisciplinary approach for studying the development of mathematical creativity of preschool children with a precarious childhood, by Melanie Münz, Germany.
- Problem modification as an indicator of deep understanding, by Mihaela Singer Florence, Romania
- Little University of Mathematics, by Laura Freija, Lithuania
- Effects of Modified Moore Method on Elementary Number Theory for Gifted High School Students: An Exploratory Study, by Hee Cho Kyoung, Korea,
- Korean Middle School Student's Spatial Ability and Mathematical Performance: Comparison between Gifted Students and General Students by Sungsun Park

These presentations ended up with multiple questions about the research conducted by different participants and the practices implemented in different countries. The need for the better connections between theory and practice become more and more clear. Following this session we ask: What research approaches can inform us in the best way? How does research methodology depend on definition of gifted chosen in the study? How research and practice can be interwoven to advance theories of mathematical giftedness and advance effectiveness of the practical projects for mathematically gifted students.

# **Characteristics of mathematically gifted students**

The fourth session of the TSG focused on characterization of mathematically gifted students. BoMi Shin from South Korea reported on a study that provided probability tasks to mathematically gifted students to investigate analogical reasoning as it emerges during the problem-solving process of students. Atsushi Tamura from Japan presented a case study about a gifted high school student in which he identified 5 prominent characteristics in thinking processes by investigating how he devised mathematical proof. Furthermore, this



study found that sharing the thinking processes of the gifted student in the classroom had a good effect on both the class and the gifted student himself.

Amaral Nuno and Susana Carreira from Portugal described analysis of creativity in the problem solving processes presented by eight students (from grades 5 and 6, aged between 10 and 11) who have participated in and reached the final phase of a Mathematical Competition. They suggested ways for evaluation of students' creativity in mathematical problem solving in a situation that includes a competitive factor and takes place beyond the mathematics classroom, which is often seen as restrictive for the development of mathematical creativity.

Brandl Matthias from Germany (in collaboration with Christian Barthel) suggested that there are two ways of selecting promising students for the purpose of fostering (in mathematics): whereas the standard procedure is to offer additional courses or material for volunteers or those chosen by the teacher, the other and perhaps more elitist - but with respect to quantitative aspects easier - way is to select the students with the best marks. Brandl argued that from a psychological perspective these ways represent two opposite sides of the causality between giftedness and assessment. One result of this investigation is the finding of strong correlations between the profiles of mathematical interests of specific subgroups that fulfill the characteristics which define mathematical giftedness.

Lecture Marianne Nolte discussed relationships between "High IQ and High Mathematical Talent!". The findings followed from the long-term PriMa-Project in the University of Hamburg. This project is a research project and a project for fostering mathematically talented children. To detect among them mathematically especially talented children demands a highly comprehensive search for talents. Marianne Nolte stressed the complexity of the evaluation of mathematical talent and stressed that search for talent poses the risk that

children may be classified wrongly as especially talented or that children's talents are not recognised.

In conclusion the following questions were raised by the group: Do we know more than Krutetskii after we perform studies on characteristics of students with high mathematical abilities? How do researchers choose their research paradigm? How do research methodologies correspond to the students' age or to a specific characteristic of giftedness that is examined? How studies on students thinking can/should inform educational practices?

The work of the group demonstrated how much is done in the field of the education of mathematically advanced students but moreover it stressed how much should be done in order to get a better understanding of the phenomena of mathematical giftedness and the effective ways of realization of mathematical potential in all students including mathematically talented ones.

## Acknowledgement

We would like to thank all the participants for their interest in the topic, all the contributors for their interesting presentations and hard work at this TSG, Team members –Viktor Freiman, Linda Sheffield, Mihaela Singer, Bo Mi Shin -- for fruitful collaboration in preparation and conducting this TSG.



# ICME-12 - Discussion Group 3 - Creativity in Mathematics Education

#### Vince Matsko

The Discussion Group 2 on Creativity in Mathematics Education at ICME-12 in Seoul, affiliated with MCG, was a great success. Co-chaired by Board members Hartwig Meissner, Emily Velikova and Vince Matsko, the group focused on issues relating to creativity in the classroom as well as training both pre-service and in-service teachers regarding effective ways of fostering creativity.

DG2 faced difficulties in communicating with ICME organizers, and as a result, participants were not able to see the proposals before the conference. As a result, the chairs decided to begin each of the two ninety-minute sessions with brief summaries of the papers delivered by their authors. These were both lively and informative, and gave the 100+ participants in DG2 a chance to be brought up to speed.

The sessions focused on the following questions, developed by the Board earlier in the year:

- 1. What does creativity mean in the process of teaching and learning mathematics?
- 2. How can we develop or stimulate creative activities in and beyond the mathematics classroom?
- 3. How might we balance mathematical skill training and mathematical creativity?
- 4. What should be done in teacher training programs at the preservice and inservice levels to foster creativity in the classroom?

(Note: a link to the DG2 website, including a full description of the Discussion Group as well as all the submitted papers, is accessible from the MCG website.)

The first session addressed the first two questions, while the second centered around the last two. The chairs subdivided the first two questions into six, so that smaller discussion groups could be formed. DG2 participants selected the question they wished to discuss, and elected a representative who summarized the discussion at the end of the session. Many participants remarked on quality of the discussions, and all were stimulated by the sharing of ideas.

The second session was rather smaller than the first, occurring on the last full day of ICME-12. As a result, participants voted to have a large group discussion rather than breaking into smaller groups. This proved to be effective – and even though participants were tired after a hectic week, it proved difficult to make sure everyone had a chance to speak. Those involved had a real passion for creating engaging activities in the mathematics classroom, and there was no shortage of ideas to share.

Of course, in discussions like these, more questions are raised than are answered. These questions came up as a response to concerned teachers truly wanting to be more creative in the classroom. Among the questions raised by the DG2 participants were:

- 1. How do we decrease pressure on students so that they are more free to be motivated and involved in mathematics?
- 2. How can we use technology to allow students to demonstrate originality, flexibility, and fluency of thought?



- 3. How can we develop creativity within a pre-service teacher's university experience?
- 4. Given we believe that *all* students can be creative, how can we create opportunities for students to do so?
- 5. How can we deliberately foster creative thinking to encourage innovation?
- 6. How can we provide accessible resources for teachers so that they may more easily bring creative activities into their classrooms?
- 7. How can we change the climate of university education departments so that developing creativity in teachers is valued and addressed in the curriculum?
- 8. How can creativity in mathematics education be made a priority at a regional or national level?

Of course none of these questions has an easy answer. But one or more of them might be suitable for a discussion forum or a special session of a conference on education. We welcome contributions to this newsletter from mathematics educators who have successfully answered one of these questions either in their classroom, or who made an impact regarding one of these questions on a local, regional, or national level.



# **Educational Innovations and History**

**Alexander Karp – Section Editor** 

# A recent history of Korean public institutions for the mathematically and scientifically gifted: From specialized science high schools to science academies

# Kyong Mi Choi and Jeong-Yoon Jang

#### University of Iowa

This article attempts to describe how some Korean public institutions for the mathematically and scientifically gifted worked in relatively recent years. One important year which is to be used as a starting point is 1969. Before that in Korea, students were differentiated based on their entrance examination performance and were assigned to schools according to their achievement levels. Sixth-grade students (the final year of elementary school) and ninthgrade students (the final year of middle school) had to take middle and high school entrance examinations respectively. In 1969, the Equalization Policy was implemented, in which elementary school students no longer took the entrance examination to enter middle schools while middle school seniors still had to take an entrance examination to attend high schools. In 1974, when these entrance exams were abolished altogether, the High School Equalization Policy took effect. Since this time, students were assigned to a school in their neighboring school districts without any considerations of their academic aptitude. These reforms were criticized that this policy took away opportunities for students to receive individual competence-appropriate education and to develop their interests and ability (see Choi & Hong, 2009 for more details).

These critiques continued in the 1970s and 80s, and eventually, among many suggestions, the idea of specialized education for the high-achieving students became prevalent (Suh & Shin, 1996). The Korean Education Development Institution adopted the idea and initiated creation of a high school specialized in science. The very first such school was founded in 1983– Gyeonggi Science High School. Early Korean Specialized Science High Schools (SSHSs) were strongly influenced by Russian schools with an advanced teaching of mathematics and science (Vogeli, 1997). In the subsequent years, other SSHSs were established in other provinces. These schools provided possibilities for students to develop their potential fully. Until 2003 when the Ministry of Educational Science and Technology (MEST) designated one of the SSHSs as the first Science Academy (SA), there were 15 SSHSs that served mathematically and scientifically gifted students of high school ages in Korea. As of 2012, there are 22 schools for the mathematically and scientifically gifted including 19 SSHSs and 3 SAs in Korea. All SSHSs and SAs have been public.

Initially, SSHSs were the only type of schools designated to educate students who were mathematically and scientifically gifted. SSHS students studied accelerated and enriched mathematics and science curricula in addition to the national high school curriculum. Since all public schools have to follow the national high school curriculum, SSHS students learn



advanced mathematics and science at an accelerated pace after completing the materials required by national high school curriculum.

Teachers in SSHSs are known as highly qualified. Generally, public school teachers are assigned and rotated every five years within a region determined by local departments of education. However, principals of SSHSs have discretion to hire teachers for their schools. The teachers are experienced and many of the teachers hold master's degrees and some have doctoral degrees. In addition to classes offered by the school teachers, students have the extra privilege of listening to lectures and working with college professors and researchers.

SSHS students are well-known for their academic achievement. A large portion of international mathematics and science Olympiad team members are SSHS students and a majority of graduates enter prestigious universities in Korea and the U.S. The SSHS graduates often continue displaying their success in mathematics and science: Many of them earned doctoral degrees in mathematics and science fields and authored recognized works.

Typically, the national high school curriculum is covered in SSHS in less than two years. During their third year, students explore advanced mathematics and science beyond the high school curriculum. Students have an opportunity to complete high school in two years and to enter Korea Advanced Institute of Science and Technology (KAIST), which is one of the best research-oriented universities for science and engineering. Those who did not follow this path are supposed to develop their interests by learning advanced mathematics and science in their final year (the third year). However, instead, some classes of the third year of SSHSs focus on preparing for the national college entrance examination.

While the practice of college entrance examination preparation at SSHSs and its original purpose of teaching and learning advanced content during the third year conflict, some educators believe that simply learning advanced mathematics content is not beneficial for mathematically gifted students and does not help to nurture and develop their interests and talents. In turn, Suh and Shin (1996) suggested increased mathematics requirements and offer mathematical activities such as discussion-based seminars and independent research to nurture individual interests.

The MEST proposed the Advanced Gifted Education Law to address criticism and suggestions, and enacted the law in 2000. According to the law, establishing special programs for gifted students and allowing acceleration and early graduation in regular public elementary, middle and high schools was encouraged. Opening gifted education centers was also supported by the law (Cho, 2000). Additionally, a new type of institution for the gifted was proposed – School Academy (SA). In 2003, the first SA – Korean Science Academy – was founded. The school was formerly known as Busan specialized science high school, one of SHSSs, and was designated the school to be converted into the first SA. It was followed by Seoul specialized science high school in 2009 and Gyeonggi specialized science high school in 2010 which became the second and third science academies. The MEST plans to convert several other SSHSs into SAs in the near future. The Korean Science Academy website explains that the school's educational goal is to 'nurture creative global scientists who will contribute to world society' (http://www.ksa.hs.kr/english/0102.php).

One major difference of SA from SSHS is its credit-hour system instead of SHSS' year-based system. Like in colleges, SA students can graduate when they complete the required credit hours while SSHS students need to stay in the school for two or three years. SA's curriculum plan, as Kwon's (2012) report describes in detail, consists of (a) academic courses, (b) creative research activities and (c) leadership activities, a total of 165 credit hours. Required and



elective academic courses include 62-credit-hour humanity courses such as Korean, social sciences, foreign languages, physical education, and music and fine arts as well as 73-credit-hour advanced mathematics and science courses. Advanced courses include elementary number theory, linear algebra, differential equations, and discrete math.

All SA students must take 30-credit-hour 'creative research' curriculum beginning at the very first year. Typically, these activities include attending a small group seminar (where students learn how to plan and conduct research projects), conducting an independent research project with a college advisor and writing a thesis on this project. One SA student has conducted a research project with two Russian specialized high school students and they co-authored a research article published in *Tetrahedron Letters*. Several other research reports of SA students' have been published in internationally recognized research journals as well (i.e., *Microbial Cell Factories*). As a part of a special 16-credit-hour 'leadership' curriculum, SA students have to be actively involved in club activities and volunteer work.

Another distinctive feature of SA's is its admission policy. Although students should graduate from middle school to enter SSHS, SA admits not only middle school graduates but also current middle school students who are recommended by teachers as talented in mathematics and science in accordance with the Advanced Gifted Education Law. In fact, a number of first and second year middle school students were accepted to SA in recent years. The students who are accepted to science academy allowed graduating without the compulsory three-year middle school education. This policy allows students displaying their talent at a young age to develop their interests and talent as early as possible.

Despite its aim to promote students' potential fully, one concern of note is the imbalance between numbers of male and female students. The numbers of female students in SSHS and SA have always been very low, about less than a third of male students. Female students were not allowed to enter the first SSHS until 1988, five years after its opening, and they were not admitted to other SSHSs until 1989. The efforts to improve this situation have been made including both changing admission processes and encouraging female students to participate in extracurricular activities such as academic competitions and camps to identify their aptitude in mathematics and science.

During the last 40 years, Korean mathematically gifted education experienced serious organizational changes. Importantly, these changes were initiated and implemented by the Ministry of Education which (differently from some other countries) played a leading role in the gifted education. However, the system briefly described here has been supplemented by private initiatives. It will be interesting to see how privately owned and operated institutions and educational resources for this population have contributed to develop the giftedness in mathematics and science.

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# **Competitions corner**

**Peter Taylor – Section Editor** 

# Mathematics Competitions and a Career in Mathematics – Some anecdotes from Austria

## **Robert Geretschläger**

#### BRG Keplerstrasse, Graz, Austria

Austria has been holding preparatory courses for its national Mathematical Olympiad since 1969 and participating in the International Mathematical Olympiad since 1970. While this may not be as long as some other countries, over 40 years have passed since those early days, and the first Austrian Olympiad participants are now nearing retirement age. It therefore seems like a reasonable time to take a look at the mathematical careers of a few of the more prominent among them, and perhaps ask whether they feel that participation in the Olympiad had any lasting influence on them.

As stated in the title, there has been no attempt made here at any kind of comprehensive statistical research of possible correlations between Olympiad participation and a future life in mathematics, although such an analysis would certainly yield some interesting results. (Much of interest can be found in [1].) The common thread of the mathematicians mentioned here is simply successful participation in the Austrian Mathematical Olympiad (ÖMO) in their early days. It is interesting to note that many of these have kept in touch with the competitions scene, and are active in some capacity, such as in the roles of problem developers or trainers. I have sent a preliminary version of this note to the people mentioned here, and the comments I received can be found at the end.

Since the early days of the ÖMO, several other mathematical competitions have been introduced in Austria. Some, like the International Tournament of the Towns or the Mediterranean Mathematics Competition, are supplementary to the goals of the Olympiad. Others, like the mathematical Kangaroo, have a different intent, not so much geared to fostering specific mathematical talent, but rather to popularising the subject in the general school context. Such competitions do, however, make it possible to recognise some students with a specific mathematical talent or interest, who can then be invited to take part in special courses, ultimately leading to individual research work in mathematics for some participants. Speaking as someone who has been involved in all of these competitions for years, it is at least noteworthy to point out that the percentage of Olympiad participants that go on to formally study mathematics or a related area actually seems to be quite large. Moreover, the number of Olympiad participants now in prominent academic positions also seems, at least superficially, to be something less than accidental.

Perhaps the most prominent position a professional mathematician can hold is as president of the national mathematical society. The last three presidents of the Austrian Mathematical Society (Österreichische Mathematische Gesellschaft, or ÖMG) all have Olympiad backgrounds. The current president, Michael Drmota, is the head of the Institute of Discrete Mathematics and Geometry at the Vienna University of Technology. As a high school student, he was a medal winner at the final round of the ÖMO in 1982 and a participant at the Austrian-Polish Mathematics Competition, and thus a member of one of the national



teams. His predecessor as president of the ÖMG, Robert Tichy, works at the Institute for Analysis and Computational Number Theory at the Graz University of Technology. Among other things, he has been Head of the Department of Mathematics and Dean of the Faculty of Mathematical and Physical Sciences there. As a high school student, he was a medal winner at the final round of the ÖMO in 1975 and a participant at the IMO. Finally, his predecessor at the helm of the ÖMG, Heinz W. Engl, has held many prominent positions in research mathematics. Among other things, he is currently Director of the Johann Radon Institute for Computational and Applied Mathematics and Rector of the University of Vienna. As a high school student, he was a medal winner at both the ÖMO and the IMO in 1971.

Among these, specifically Robert Tichy has been very active in supporting the mathematical Olympiad by organising and occasionally lecturing at mathematical training sessions for students of all levels as well as teachers.

Another prominent Austrian mathematician worth mentioning in this context is Christian Krattenthaler. A full professor and vice-dean of the faculty of mathematics at the University of Vienna, he is also a recipient of the prestigious Wittgenstein prize. In high school, he was a prize winner at the ÖMO in 1977 and a participant at the IMO.

From 1993, some areas of Austria (Graz in particular) have taken part in the International Tournament of the Towns. As a supplement to the Olympiad, this competition has helped to give many students additional competition experience, but also a wealth of mathematical material to think about and advance their knowledge. One of the most successful participants was Clemens Heuberger. Now a full professor of discrete mathematics at the Alpen-Adria-Universität Klagenfurt, he was the first Austrian to be invited as a participant of the summer seminar of the Tournament of the Towns, but also a medal winner at both the ÖMO and IMO in 1991, 1992 and 1993. Beside his work in research and teaching, he is also very active in the Austrian Olympiad as a trainer on the national level and as coordinator of the problem group for the so-called "beginners" competition. While still in Graz, together with Tichy he was responsible for attracting many students at the TU, who were former Olympiad participants themselves, to work in student training at all levels.

Quite interesting is also the trajectory of the mathematical careers of some former ÖMO participants that have led to prominent posts at universities in other countries, where they in some way continue their support of the national Olympiads in their new homes as well as Austria:

**Theresia Eisenkölbl** is a mathematician at the Institute Camille Jordan of the Université Claude Bernard in Lyon, France. As a high school student, she won gold medals at the ÖMO in 1992, 1993 and 1994. At the IMO, she won medals in all three of these years, capping her achievements there with a perfect score in 1994. She is currently active in problem setting for the Austrian Olympiad, and is also associated with the French Olympiad.

**Gerhard Wöginger** is a professor at the Department of Mathematics and Computer Science of Eindhoven University of Technology in the Netherlands. He was a medal winner at the ÖMO in 1981 and 1982 and an IMO participant, and is still very active as a problem setter for many competitions. Several of his problems have made their way onto the papers of the Mediterranean Mathematics Competition in recent years, and he has had problems on the short list of the IMO in each of the last 5 years.

**Stephan Wagner** is an associate professor at Stellenbosch University in South Africa. He was a four-time medal winner at the ÖMO (from 1997 to 2000) and a three-time medal winner at the IMO (from 1998 to 2000). He is also still involved in the organisation of and



preparation of students for mathematical Olympiads both in Austria and South Africa, and was leader of the South African IMO team in 2011. He is also preparing to play an important role at the IMO to be held in South Africa in 2014.

The mathematicians mentioned in the note were asked the question "Do you feel that your participation and success at the ÖMO had any influence on your future career?" The following answers were received (in alphabetical order):

#### Michael Drmota

"Actually the experience of participating in the Mathematical Olympiad was quite influential for me. It was definitely the basis for the decision to study mathematics and the practice of solving mathematical problems (and also the knowledge of formal concepts) was very helpful during my studies. When it goes to mathematical research one needs definitely more than that, nevertheless the "technical skills" from the Mathematical Olympiad form a solid basis."

#### Heinz Engl:

"The extra courses we had for preparation for the Olympiad completely changed my view about mathematics I had from the ordinary high school curriculum. Without these courses and the successful participation in IMO, I might not have studied mathematics at all."

#### **Clemens Heuberger:**

"Taking part in the Olympiads made it clear to me that I really should study mathematics. It also gave an advantage at the start of my studies because formal mathematical reasoning as well as some endurance were skills which I had already acquired during Olympiad training."

#### **Christian Krattenthaler:**

"I do not think that my \*success\* at the ÖMO had any influence on my future career. BUT: It was the participation in the ÖMO, and here particularly the preparation courses at my school and in Raach (i.e. the two and a half week seminar leading up to the final round; RG), which stimulated my "appetite" for mathematics, and thus it has a lot to do with my future career as

"enseignant-chercheur" (as the French say) in mathematics. The problem-solving character of the Olympiad may not be appropriate for everybody with a mathematical interest; for me it was exactly the right thing to enter and delve into the beauties of mathematics.

#### **Robert Tichy:**

"The mathematical Olympiad was certainly a significant reason for me to choose mathematics over physics as an academic field of study."



#### **Stephan Wagner:**

"The Olympiad certainly had (and continues to have) a major influence on my life. It was the participation in those competitions that made me realise that mathematics is the right career choice for me - without them, I might have studied physics or engineering (or even something completely different) instead. This is why I like to stay involved in Olympiad training - it gives me the opportunity to attract talented young people and show them how rich and beautiful our subject is."

#### Gerhard Wöginger:

"This had strong impact on my future career: it triggered my interest in mathematics, and the knowledge that I had built up for these high school competitions made my first year at university fairly easy. By the way, my co-authors Yossi Azar (Tel Aviv) and Jiri Sgall (Prague) both attended IMO 1981 in Washington together with me; perhaps we met there for the first time, but we do not remember talking to each other.

#### Reference

 [1] Friedrich Oswald, Günter Hanisch, Gerhard Hager (2005) Wettbewerbe und "Olympiaden": Impulse zur (Selbst)-Identifikation von Begabungen, LIT Verlag, Berlin-Münster-Vienna-Zürich-London, ISBN 3-8258-7604-7



# **Problem Corner**

**Romualdas Kasuba - Section Editor** 

# **TETRADS – IT IS NOT SO EASY**

# A. Cibulis, J. Čerņenoks

University of Lithuania, Riga

#### Abstract.

In this paper a short review on tetrads is presented. Attention is paid to pupils' achievements.

#### Introduction

A **tetrad** is a plane figure made of four congruent shapes, joined so that each one shares a boundary (of positive measure) with each. We can find some tetrads as well as information about the first contributors in the Martin Gardner's book [1]. One can find the next contribution in [2], [3]. In accordance with [1]: "Michael R. W. Buckley, in the *Journal of Recreational Mathematics*, 8 (1975), proposed the name tetrad for four simply connected planar regions, each pair of which shares a finite portion of a common boundary." Let us note that in the Russian translation (1993) of the book [1] the reference to this journal contains the scribal error in the year (1985). In fact these are all sources on tetrads that we were able to find. As to the mathematical Olympiads (MO) of Lithuania we are aware of only one contest problem concerning tetrads, moreover, this problem seems to have been derived from [1]: "On the square paper draw four equal hexagons the sides of which go along the grid lines and so that each hexagon shares at least 1 mm length boundary with each." (Open MO of Lithuania, 2002) As the solution the following construction (Figure 1) has been given:



The tetrad shown in Figure 1 has been made from four equal 9-ominoes. Let us remind the reader that a **polyomino** is a plane shape formed by joining unit squares edge to edge; **n-omino** is a polyomino having exactly n unit squares. We will consider only tetrads made from polyominos in this paper. Starting from this tetrad we firstly will formulate some questions that would naturally arise to curious pupils: Does a tetrad made from 8-ominoes exist? Has a tetrad always a hole? If so, what is a the minimum hole? Try to give the answers before the further reading!



#### The first 4 problems

- Problem 1. Find a tetrad made from 8-ominoes.
- Problem 2. Does a tetrad made from 7-ominoes exist?
- Problem 3. Find a tetrad without holes.
- **Problem 4.** MCG group consists of 16 members. To symbolize friendliness and compatibility of this group it is necessary to construct a tetrad consisting of four 16-minos being also 16-gons.

#### **Solutions and comments**

- **1.** Four tetrads found more frequently by pupils has been shown in Figure 2. There are 14 tetrads made from 8-ominoes, this has been checked by a computer, see e. g. [3].
- **2.** No. This answer has been obtained by a computer. Conceivably some gifted student will find a short proof of this fact.





**3.** As far as known the first full (holeless) tetrad for a polyomino has been found by Scott Kim [1], see Figure 3. This tetrad consists of 10-gons and fits in the rectangle 8 × 10. A simpler full tetrad consisting of 8-gons and, moreover, fitting in the smaller rectangle 7 × 9 has been shown in Figure 4. George Sicherman is the first to have stated the smallest holeless polyomino tetrad uses 11-ominoes by a computer [3]. The independent and wider analysis was done by Juris Černenoks in his Master Thesis (2012). It turns out that there is only one full tetrad made from 11-ominoes (Figure 5). As to 12-ominoes there are only 10 full tetrads (out of all 5,648 tetrads). Try to find all of them. It is not so easy!





4. It is not so easy to find such a tetrad (Figure 6), especially a full tetrad (Figure 7).



#### The tetrad problem of the year of 2012.

For each pentomino find a minimal tetrad including this pentomino as the unique hole.

This challenging problem (proposed by A. Cibulis in 2011) consists of **12** subproblems becauce there are 12 pentominoes. In this year "The Little University of Mathematics", organized by A. Liepa's Correspondence Mathematics School, resumed the work. The 12 popular-science lectures (2 lectures every day) was delivered to pupils of Lithuania. This problem on tetrads was offered also in the lecture "On Some Peculiarities of Mathematics, and Puzzles" by A. Cibulis, see [4], as one of the appropriate problem for pupils in their research works as well as for science talents search. Two striking talents in tetrad constructing were discovered. Aleksejs Popovs (Form 8, Riga) found the tetrads shown in Figures 8 – 9. Olegs Matvejevs (Form 10, Riga) found tetrads including the pentomino: I, N, P, T, U, and V, see Figure 10. Moreover, he was able to find the method for generating tetrads with the prescribed hole. The idea has been illustrated for the pentomino X in Figure 11. It is obvious that the pentomino X can be replaced by any other pentomino. This method does not give the minimal tetrads. Does effective method for generating minimal tetrads exist? Unfortunately we do not know the answer.



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Figure 11.

O. Matvejevs succeeded to find three minimal tetrads having the hole I, N, and T respectively, see Figure 10. Pupils of Andy Liu (Canada), Hans and Daniel, were able to find two minimal tetrads including P and Z pentomino respectively, see Figures 12–13. Are you able to find tetrads for pentomino F, L, W, X, Y and the smaller tetrads for pentominoes U and V?

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# Some other remarkable tetrads

Anastasija Jakovļeva (Form 10) in her contest paper [*Analysis of Tetrads*, Riga, 2012] found very *nice* tetrads made from the parts of the square (Figure 13), and proved that every square can be the unique hole for some tetrad. The idea is simple: the following picture (Figure 14) serves as PROOF WITHOUT WORDS:





## Solution of the tetrad problem of the year of 2012.

In the master thesis (2012) of Juris Černenoks special computer programs have been elaborated to generate all polyomino tetrads with the area not exceeding 60. As a consequence a lot of minimal tetrads have been found. Results for minimal tetrads made from pentominoes are given in the Table 1. The number in the second row indicates the smallest n-omino generating the corresponding tetrad. Only for pentomino X the number 17 might be reduced to 16. Are you able to find tetrads with characteristics as in Table 1. It is not so easy!

	F	Ι	L	Ν	Р	Т	U	v	W	x	Y	Z
10	15	10	12	15	10	15	15	15	13	17?	15	12

Table 1.

## References

[1] M. Gardner, Penrose Tiles to Trapdoor Ciphers, Freeman, 1989.

[2] http://demonstrations.wolfram.com/Tetrads/

[3] <u>http://userpages.monmouth.com/~colonel/tetrads/tetrads.html</u>

[4] http://nms.lu.lv/mmiu/mmu\_11\_12.shtml



# Solution of Trapezoid Problem (see, MCG Newsletter 2)

#### Presented by the author: Andrejs Cibulis,

University of Lithuania, Riga

## **Exercise 1**

Prove that for any trapezoid the intersection point *O* of the diagonals is also the middle point of the line *EOF* parallel to the basis, see Figure 1.

## **Exercise 2**

Prove that  $x = \frac{ab}{a+b}$ , see Figure 1.



# **Solution 1**

From the similar triangles  $\triangle ABD \sim \triangle EBO$ ,  $\triangle ABC \sim \triangle AEO$ , see Figure 1, it follows:

$$\frac{a}{x} = \frac{u+v}{u}, \ \frac{b}{x} = \frac{u+v}{v} \implies au = bv \implies \frac{a}{x} = \frac{bu+bv}{bu} = \frac{a+b}{b} \implies x = \frac{ab}{a+b}.$$
  
The same formula holds also for *y*, i.e.  $y = \frac{ab}{a+b}$ .

## **Solution 2**

From the similar triangles  $\triangle ABD \sim \triangle EBO$ ,  $\triangle ABC \sim \triangle OGC$ , see Figure 2, it follows:

$$\frac{a}{x} = \frac{u+v}{u} = \frac{b}{b-x} \implies ab - ax = bx \implies x = \frac{ab}{a+b}$$





The same formula holds also for *y*, i.e.  $y = \frac{ab}{a+b} = x$ .

As to students' solutions (see Newsletter 2) the explanation is simple. These solutions are correct (it is true, but not so economic solutions as the above given ones), except the final step in which students have hurried with the conclusion a = b from the equality (a-b)(x-y) = 0. In fact this equality implies that x = y. There are other ways to determine *EF*, see, e.g. [1], [2].

**Remark**. *EF* is the harmonic mean of *a* and *b*:

$$x + y = x + x = \frac{2ab}{a+b} = \frac{2}{\frac{1}{a} + \frac{1}{b}}.$$

The notion "harmonic mean" was introduced by Hippas of Metapont (ca. 450 BC, see [3]. **Corollary**. How to construct a median without a compass?

Let *APD* be an arbitrarily triangle,  $B \in AP$ ,  $BC \mid \mid AD$ , and *O* is the point of the intersection of *AC* and *BD*, then the medians of the triangles *BPC*, *EPF*, *APD* lie on the straight line *PO*.

#### References

- [1] http://planetmath.org/encyclopedia/HarmonicMeanInTrapezoid.html
- [2] <u>http://jwilson.coe.uga.edu/EMAT6680Fa08/Wisdom/EMAT6690/Investigate%20</u> <u>Means/Means\_njw.html</u>
- [3] <u>http://www.cs.cas.cz/portal/AlgoMath/MathematicalAnalysis/Inequalities/</u> PythagoreanMeans.htm



# Might it be so that being perfect sometimes indeed might only mean being simple or as short as possible?

## Romualdas Kašuba

#### Vilnius University, Lithuania

Being perfect in problem solving or being effective in problem posing is often related to the rare ability of remaining as curt as possible when speaking or convincing when presenting fairy tales. Any good or exciting fairy tale might only be expressed in a few words but once you hear it you can't forget it. Something stays with you without you even realizing it. The whole process is similar to what might stick with you from what you've seen and enjoyed throughout your life and is the matter of your fantasy, skills, will, temperament and imagination.

Let us again cite some suitable poetry, which while being short and clear, can give you a taste of feeling and flavor of understanding many things, notions and situations that might be of considerable value and importance for so many of us.

"Tis the voice of the Jubjub! Keep count. I entreat; You will find I have told it you twice.'Tis the song of the Jubjub! The proof is complete, If only I've stated it thrice."

The ability to be and to remain simple when doing something will always remain among the first of arts or anything else you might be lucky enough to see and enjoy in your life.

Let us remain optimistic – math, with all its arithmetic skill and abilities, remains much more optimistic than some of us were able to imagine while staying at school and other similar places of serious importance.

Still remaining optimistic, we have the right to cite yet a few other lines of immortal poem by Lewis Carroll whose name is "Hunting of the Snark".

The Butcher would gladly have talked till next day, But he felt that the lesson must end, And he wept with delight in attempting to say He considered the Beaver its friend.

This is also tightly linked with the art of problem solving and with the pleasure of understanding how all it works.

Allow us to tell you something about their origins later, sometimes with their hints and sometimes with their solutions.

# Problem 1 (issue 2).

This was a problem in which we were challenged to find the smallest natural positive integer that admits two different splits into the sum of three different addends (we assume that all six of these addends participating in those two splits are all different integers).

When solving it with pleasure, we might wish to demonstrate you two things:



(A) Firstly, to present you some natural integer, which can be expressed in two different ways as a sum of three different natural summands. For the sake of avoiding any confusion, as told, we demand that all the six summands are different.

After such presentation you will clearly understand that with our next step we are going to ask you what is the smallest among all such natural numbers.

Because it is clear that very small integers already do not possess such capacity, e.g. such a capacity does not possess the number 6.

We will use the common fact that in any set of naturals there always is the first (or the smallest) integer.

So now we want to determine what is the smallest among all of such positive integers and explain why this is indeed the case.

As told, there are many natural integers that may be represented in one way and in another as a sum of three different addends and all six addends mentioned in these two representations of one and same integer are all different.

We might even speak about the clouds in the sky that have been seen in three groups. The wind might blow and the same clouds may be split in another three groups – with none of them lost.

As an example take the gathering of 33 clouds being seen in the skies in three groups with 16 in the first, 10 in the second and 7 in the third. After "the wind has blown over the ocean" the same 33 clouds were being seen in the same sky but already in other three groups – one group containing 30, another 2, and the third – 1, that is, single cloud.

Where are the boundaries?

The first observation is that with 10 clouds and no wind blowing, as well as no efforts in our mind, could ever perform such a trick. Indeed, let us regard the 6 smallest natural numbers or the first 6 friends of all human beings or first initial integers

#### 1, 2, 3, 4, 5, 6.

Their sum is clearly 21 which more than twice as big as 10. That already guarantees that the no wind could do anything with 10 clouds.

The second observation is that already with 11 clouds the wind might be successful in its blowing, as it can be easily seen from the example below:

#### 11 = 7 + 3 + 1 = 2 + 4 + 5.

The third observation is that we must be careful and add that if the wind is successful with the sample containing N clouds than, sooner or later, the wind will be also successful with N+1 cloud as well.

## Problem 2 (issue 2)

We knew that it would not be very difficult for you to split the plate of chocolate  $5 \times 5$  into 7 rectangular pieces. We were so democratic, that even the  $1 \times 1$  part of chocolate was regarded as one of such possible pieces.



We also told you that there is more than one solution and now we are going to present you one of them.

In our solution, the entries belonging to the same rectangular piece are all marked with the same number from 1 to 7 – and only in such a case.

1	3	5	6	6
1	3	5	6	6
1	3	5	7	7
1	4	5	7	7
2	4	5	7	7

So, you see that this is indeed one of the few possible answers. You could even be tempted to try to demonstrate that in every other split there will always be all these rectangular pieces.

Let us make the list of all rectangles we used. These are 1 x 1, 1x 2, 1 x 3, 1 x 4, 2x 2, 1 x 5, 2 x 3.

(Remark: The rectangular 1 x 2 will be regarded to represent the same rectangular shape as 2 x 1, etc.)

Would you try?

#### Problem 3 (issue 2).

We asked you to solve "the right rebus"

As you might well remember, first we asked you to present us an arbitrary solution.

And many of you surely were able to deliver some of the possible solutions:

2970 + <u>2386</u> 5356

Then we asked about solutions of that where

ONE,

is regarded as the 3-digit number, takes the smallest possible value. In the previous solution, as we see now,

ONE = 956

Again, you'll delivered the solution about the smallest ONE so quickly (together with the following example):



1960	
+1472	
3432	

We believe you might have noticed that the smallest

is not so small.

We do not dare to ask how you found the solution. We believe our readers, who are all real or potential solvers. We are very glad to inform you that this problem is created by a famous composer of mathematical problems, Igor Ivanovich Voronovich, who for more than 20 years is the leader of the Belarus team at the International Math Olympiads. Together with Voronovich other famous problem composers, such like S. A. Mazanik, another leader of Belarus IMO team, V.I. Kaskevich and E.A. Barabanov shouldn't be omitted. And, without any doubt, the name of V. I. Bernik should occur.

Some others of proposed problems in that chapter are also of Belarus origin, e.g., problem 5.

## Problem 4 (issue 2)

This problem is in some way converse to one we proposed in the first issue of our Bulletin:

Is there such a natural integer with 2 as its first digit and such that, if we make that 2 to be already its last digit, then the number we get is exactly 2 times smaller than that initial number.

Being converse it sounds like this:

Is there such a natural integer with 2 as its first digit and such that if we make that 2 to be already its last digit, then the number we get is exactly 2 times bigger than that initial number.

The difference of these two problems is exactly one word: smaller-bigger. And sometimes one word might mean very much in the sense that by changing only one word we might also change rather attractive word of answer "yes" to much less sympathetic word of answer "no".

"Yes" in that case means changing

210 526 315 789 473 684

to

#### 105 263 157 894 736 842

becoming in such a simple way 2 times smaller.

And attempts in the similar way fails, because the number whose digits from the left to the right respectively are

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cannot make "only a half" of the number whose digits respectively are

This might be established in many ways, even by writing some simple prosaic equations.

#### Problem 5 (issue 2)

The first arithmetical dialog of Robinson Crusoe with the Man Friday was described in the annals of that Island this way:

First, Robinson Crusoe wrote down a 3-digital number and asked his true servant and assistant to write down some of its divisors.

After careful consideration, the Man Friday equally carefully wrote on the sand (possibly with the finger) the following numbers:

7, 10, 27, 35, 45 and 63.

It was the resolute voice of Robinson, who announced that from all these 6 numbers, written on the sand by the finger of Man Friday, exactly 4 from 6 indeed are the divisors of that 3-digital number.

The question was establishing what that initial number was.

We are going to present you the solution, that also might be found, as told, in the wonderful problem solving books from Belarus.

We kindly ask our reader to let us present the solution of that last problem in the form, as it would be made below.

"Solution", or let us try to be of some use to that famous Man Friday and let us consult him from the arithmetical point of view. Somehow we have got an impression that Man Friday will listen to us carefully and together we will all experience some joy by understanding simple things, that might be quite challenging and interesting at the same time.

So, first let us try to examine more carefully these 6 numbers, 4 of which are indeed divisors of some 3-digital number (while the remaining 2 are not). We can easily decompose all of them into the product of their prime divisors – this is a common thing in math – and also in arithmetic.

Then the line

#### 7, 10, 27, 35, 45, 63

when split into prime factors will look like it is written below:

#### 7, 2 • 5, 3 • 3 • 3, 5 • 7, 3 • 3 • 3 • 5, 3 • 3 • 7

And we ask the following question: what could be told for sure about the divisors of the number that we all, together with Man Friday, would so eagerly like to find? The smallest divisor that divides more than one from mentioned 6 possible divisors of the number, for which we are now looking for, is

3.

The number 3 divides 3 of numbers mentioned. There are more such divisors dividing three of mentioned integers. Also, 5 do divide three numbers in our list, as well as 7 do – and so is 9.



So we've found the numbers 3, 5, 7 and 9 dividing three numbers from the Man Friday's list. Do they divide the number that we all are looking for?

Yes, each of them does, because, if 3 doesn't divide our 3-ditigal number we are looking for, then three numbers from the list, or 27, 45 and 63 also do not divide our 3-digital number – and this is indeed so, because, as mentioned,

3

divides all 3 of them. But one of them must then be a divisor of the number we all so eagerly looking for.

The same remains valid for the numbers 5, 7 and 9. Each of them, exactly from the same reasons, must also divide our wanted 3-digital number, because otherwise there are other three numbers from the list that again were not the divisors of our 3-digital number, which all of us would so eagerly like to detect – and this would be and already is the contradiction.

So 3, 5, 7, 9 are the divisors our 3-digital number. Now, because of 5, 7 and 9 have no mutual common divisors, so, just alone for that reason, our number, for which we all are so eagerly looking for, must be immediately divisible also by their product, or by the number

 $5 \cdot 7 \cdot 9$ 

which is exactly

315.

So our wanted number, that we all are trying to find, is without any doubt is divisible by 315. Being 3-digital and being divisible by 315 in the same time, it could only be

#### 315, 630 or 945.

The last two possibilities are impossible, because of 630 is already divisible by 5 numbers from the list, made by Man Friday – or, by all of them, but 27.

Also 945 is divided by 5 of 6 numbers - or by all of them, but 10.

So only the number 315 is divisible by 4 numbers from 6 numbers from the Man Friday's list. Being 3-digital

315

is the answer of the problem.

So, 315 is the desired number, or, the number, which we all together with the Man Friday were so eagerly checking for. And, as it often happens after eager attempts, we've found it.

## Problem 6 (issue 2)

An excellent mood of the Owl, who always had a goal in mind, as well as the outbursts of arithmetical fantasies of the White Horse usually were not so easy to foresee. Still, when they took place, the Owl rather often started shouting not just a single question, but the entire series of them. Then even such an absent-minded dreamer like the Hedgehog in the Fog or the Grizzly Bear were completely aware that everything was well in the society of owls and even horses.



You may easily start solving an actual series of questions yourself. Question that we've heard, being on every place in that scientific forest. It sounded as follows (it also ought to be mentioned that the answers were always supposed and expected to be presented to the Owl immediately and personally).

- (A) Is it possible to detect such an integer with the sum of digits not divisible by 6?
- (B) Is it possible to detect some three consecutive natural integers with the sum of digits also not divisible by 6? That non-divisibility was pre-assumed separately in all three of cases. The request for immediate answer was even more categorically expressed than it usually was.
- (C) Is it ever possible to hope that there are six such consecutive natural integers with the sum of digits again not divisible by 6 in any of all six cases?
- (D) And in general, how to deal with the following extreme general question of that resolute Owl: at most, how many consecutive natural integers are there with sum of digits not divisible by 6 in any of cases?

The solution leads us to the following auxiliary, but very much related problem, or to attempts to answer the question: how many there are such integers – with the sum of digits not divisible by 6 – are there among 10 numbers

It is clear that among any such 10 numbers there might be at most 5 of such numbers, for instance, among 10 integers

such are the last 5 of them or

Now we would eagerly expect that after that was done, we could get the same situation with another 5 integers with the sum of digits not divisible by 5 – but now among the first 5 of them. It would make then that longest possible such series contain 10 consecutive integers with the sum of digits not divisible by 6 in any of cases.

But in our concrete example we see that the initial first number in that our new tenth, or

30

(with 3 as sum of it digits) makes that our series wouldn't last so long as expected – and so it will contain only three digits 30, 31 and 32, instead of expected 5, making longest series known containing "only" 8 numbers so far:

Now we will explain why that series could be longer. From the technological point of view this is related to the fact the number 6, mentioned in our condition, is divisible by 3. And the divisibility by 3 of the sum of digits is much deeper connected with the divisibility by 3 of the number itself – because, as known, the divisibility by 3 of the number itself and the sum of digits are simply equivalent.

So now, when you have the five numbers on the end of the 10 integers

....0, ....1, ....2, .....3, .....4, ....5, .....6, .....7, .....8, .....9



then the last of them has the sum of digits 6k + 5, so the rest of sum of its digits, when dividing it by 3, will be 2 and this means that the rest modulo 3 when diving the number itself by 3 will again be 2. And, if so, then the first integer in the coming new tenth will be divisible by 3, so the sum of its digits will also divisible by 3 and can't be no more of the form 6k + 1 but only 6k + 3 (in the best of cases!). And when being of the form 6k+3 it will produce another 3 wanted consecutive integers making that longest series possess the length at most 8.

And this makes the crucial difference, say, with the case when the divisibility of sums by 5 (instead of 6) would be required. Because now we could have not more than 4 such numbers between any 10 in the end of

...0, ....1, ....2, ....3, .....4, ....5, ....6, .....7, ....8, .....9

and then hope to get immediately another 4 in the very beginning of the new coming tenth. And all these expectations might come true as the example with the following 8 numbers

demonstrates.

And now we would like to propose the new problems for our readers to solve. In order to make an additional attempt to refresh our readers, we propose and promise to mention the name of the first reader who will contact us with the right answer together with some short comments about how he got it. This is valid for any of the problems mentioned below.

#### New problems

1. The brave Robinson Crusoe, already a skilled Captain, is sailing in the billowy Ocean of Fundamental Numbers and hopes to reach the Island, where only the integers having exactly 100 digits, divisible by 100 and the sum of digits being also 100, and only these, might find its place of residence. Preparing for that serious event, that is, for the Landing on that magic place, our brave Captain first of all expressed two categorical wishes: one of them modest and another one – not at all that modest as that previous one – the crew found even to be slightly fundamental or even going to some extremes.

The modest wish is to meet any inhabitant of that Island.

The slightly extreme wish was, of course, to meet the President of that Island in person. Needless to say, but should be mentioned, that the President of that honorable Island is the smallest integer among them all. In other words, on the hat of Mr. President there is the signature: I'm the smallest integer, having not only 100 digits, divisible by 100 as well – and also 100 is the sum of all of my digits. It might be added for the light gossip claimed and repeated, that even the Nickname of Mr. President itself once and for long have been: He-Who-is-really-fond-of-zeroes-as-its-Digits.

2. If we have 30 identical wages all of them of wage 1 then we can easily rearrange them in 5 groups of the same wage and also in 6 groups – also of the same wage. It's no wonder because we still remember more than very well that

 $5 \cdot 6 = 30$ 

And we fell pretty well that all these equalities and possibilities remains to be very close one to another. We just can take 5 groups each containing 6 1's or



#### 1, 1, 1, 1, 1, 1

or we might take 6 groups consisting of 51's

1, 1, 1, 1, 1

in each.

This is somehow slightly too simple and because of that so let us now for a while be more modest and allow the system of wages contain wages of any weight – why must they always be of the same weight? Let us demand only that the system of wages could be rearranged into 5 parts of the same wage and also into 6 parts - also of the same wage.

Now we might expect that we could that with "much less" than 30 wages. So for instance we might take 15 wages

1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3

and do the same as well because we can have 5 groups 1, 2, 3 of weight 6 and 6 groups of weight 5: one 1, 1, 1, 1, 1, 1 and another 5 groups consisting from wages 2 and 3.

This is remarkable progress – we considerably reduced the number of wages, because 15 looks modestly comparing it with 30. Still, we have a feeling that we could be much more effective in reducing the number of wages.

We kindly ask you to go on reducing the number of wages and proving – in the very end – that you've got the minimal system.

We will be waiting on your solutions.

3. Consequent integers. The number is said to be consequent if it possess at least 3 digits and can be represented as a sum of two or more consecutive integers. So, for instance, 101 is consequent because 101 = 51 + 52.

Consequent is even 102 because 102 is divisible by 3 and so

 $102 = 34 \cdot 3 = 34 + 34 + 34 = (34 - 1) + 34 + (34 + 1) = 33 + 34 + 35.$ 

We ask you very confidently: would you ever believe that a 3-digital number could be non-consequent?

If so, then we are eagerly waiting for a proof of such a number.

4. There is a token in the left most square in the lowest row in a 5 x 5 square. Hedgehog in the Fog and the Grizzly Bear in turn are going to move it into the neighboring – that means, sharing the common side – field. Hedgehog makes his move first. The player who isn't able to move the token into such field where that token hasn't been before, loses. Is it possible that any of these players possess a winning strategy – that is, that he always wins no matter what the other one does and how in such a case he could proceed?

We are again ready to wait for answers – who is able to win – if any are able to do that – and how then he could proceed?

Winning strategy - what could be more exciting?

5. Trapezoid in any quadrangle of which two opposite sides are parallel and another two are not.



It is known and given that the diagonal of some isosceles trapezoid – isosceles means that these non-parallel sides are of the same length – divides it into two isosceles triangles.

Is it then indeed possible – knowing only that much – already to detect how big the angles of that initial isosceles trapezoid are?

Geometry is probably not the easiest among the serious arts – but we hope and believe that our reader will solve our problems without any remarkable difficulties and will present us the answers.

It is enough to present a solution of at least one of proposed problems.

We would like to add a problem that is from Sankt-Petersburg, and, as you will see it, you will enjoy it immensely.

6. Tom and Jerry are immensely fond of doubling any of the 10-digital integers. The only condition that they always keep is that all these 10-digit numbers before doubling as well as all numbers after it cannot contain any zero as its digit.

And then Tom and Jerry eagerly wish to know at most how many times the product of the digits of the number after doubling can be smaller than the product of digits before that doubling?

As told and mentioned, we are eagerly waiting for your answers, explanations, proposals and comments.

Please send your solutions to *Romualdas Kasuba* [romualdas.kasuba@maf.vu.lt] Solutions will be, as usual, published in the next issue of the MCG Newsletter!



# **Creative news in mathematics**

# **Mathematical Creativity of the Teacher**

## **Emilia Velikova - Section Editor**

University of Russe, Bulgaria

## **A Commentary**

Teoh Poh Yew is a famous scientist all over the world with her workshops stimulating students' mathematical creativity. At congresses and conferences around the world on mathematical education you can find a small beautiful woman in a national Malaysian costume who teaches mathematics so that all are enchanted by the power of the mathematical creativity.

She says that she is an "excellent practitioner" because she can stimulate students' mathematical creativity by one problem only. But reaching this goal is a result of deep theoretical analysis of mathematics as science, a long period of pedagogical experiments directed at the possibility of the mathematical creativity for increasing motivation for learning and developing students' knowledge and skills. Teoh Poh Yew is a creator who researches the impact of methods of presenting the mathematical knowledge to students by creating and using original learning tools with very high quality.

Teoh Poh Yew has published three books:



Amazing Mathematics Card Tricks



Teaching Children Mathematics the Fun & Magical Way!



Mathematics Magic & Craft (Plus DVD)

The <u>Wise Magic Cards</u> for mathematical games are based on mathematical problems with some interesting solutions.



Even the business card of Teoh Poh Yew is created as a creative mathematical "magic tricks".





The books of Teoh Poh Yew are very useful both for teachers and for learners because:

- 1. The problems are presented with beautiful creative solutions.
- 2. Different kinds of problems are involved in increasing order of difficulty.
- 3. General methods of instruction for stimulating students' mathematical creativity with the help of problems are presented.
- 4. For every problem, there is a case with instructions for teachers, which has been tested by the author in a real educational environment.
- 5. The graphic design of the teaching tools is developed by professionals according to the needs of the students' age and the learning process.
- 6. A creative approach is used when presenting the teaching materials in a box or as a book, which requires the individual involvement of the user.
- 7. The characteristics of giftedness are presented and they are useful for teachers and parents.

Teoh Poh Yew is an active member of the international group for Mathematical Creativity and Giftedness. She is identifying and supporting development of mathematical giftedness by:

- Organizing creative training workshops for students and for their parents;
- Organizing mathematical groups of students of different ages for joint learning of mathematics;
- Using a training process that stimulates creative thinking in a fun learning environment that brings satisfaction to students;
- Developing new educational tools;
- Disseminating information about the power of the mathematical creativity

Teoh Poh Yew sent for our newsletter one creative mathematical trick with the following acknowledgement: "I do not know who the original inventor of this trick is. Therefore, I am unable to give due acknowledgement. But I would like to thank Prof. Avi Berman for sharing such a wonderful trick with me". We are glad to present this trick herein.



# **Boggling Binary Card Trick**

This amazing trick requires you to recruit a student to be your partner in magic. Together, you are set to dazzle your audience!

#### Procedures

- Get your audience to pick any five m-Wizy<sup>™</sup> cards (or normal playing cards) and hand them over to you. After looking at the cards, select one and place it faced down. Then, line the other four below it. By looking at which of the four cards are faced up or down, your partner in magic will be able to immediately reveal the number, shape and color of the closed card above. How is this possible? It is the magic of the Binary System!
- 2. Now, based on the Binary System, each of the four cards (from your right to left) represents the values 1, 2, 4 and 8. This means that the first card represents 1, the second represents 2, the third 4 and the fourth 8.



#### If you flip this card over, you see "5 - red circle".



- 3. And the secret lies herein: your partner's job is to add the represented values of each opened card. In other words, by placing the first and third cards open faced, you are indicating to your partner that the secret card bears the number 5 because 1 + 4 = 5 (101<sub>2</sub>). Note that every closed card indicates zero and every open card represents 1.
- 4. Let's take another example: by placing the third and fourth cards open faced, you are "telling" your partner that the number is 12 because 4 + 8 = 12 (1100<sub>2</sub>).

## *Newsletter* of the International Group for Mathematical Creativity and Giftedness







If you flip this card over, it is actually 12 blue triangle.



- 5. So how do you indicate to your partner the shape and color of the special card? By making sure that the first opened card (from your left) is one with the same shape and color as the close card on top!
- 6. Let's give this a try. By looking at the cards laid out below, what do you think the special card's number, shape and color are?







It includes 13 green squares because 8 + 4 + 1 = 13 (11012) and the first open card from the left is a green square card.

Suggestions for Development of Mathematical Thinking and Creativity

- 1. Repeat the trick again and again and prompt the students to figure out how the trick works. Let them discuss among themselves and give some hints when necessary. One useful hint can be by showing them example of sets having the answer 1, 2, 4 and 8 respectively.
- 2. After the students have figured out how the trick works, let them become magician and perform the trick for their partner to reveal the answers of the top close card. In order for our students to fully understand the principle behind the game, I suggest we ask them to take 4 cards having different shapes or suits (circle, rectangle, triangle, square or heart, diamond, club and spade as in normal playing cards) plus one more card of any shape/suit. If not they might get the arrangement right by chance, without having to consider which is the suitable card to be used as the closed card on top.
- 3. Ask them questions like, "Is it possible to perform this trick using any 5 cards?" and "What are the rules that need to be changed if some cards have a bigger value such as 20?".
- 4. Ask them, "What if your audience handed these 5 cards to you one circle card, one rectangle card, one triangle card, one square card and one blank card? And your partner is aware that there are some blank cards in this stack of m-Wizy<sup>TM</sup> cards."
- 5. Encourage the students to modify the game based on other numbering systems such as base 3.

#### \*Note:

In my country, playing cards are associated with gambling. Therefore they are generally not allowed to be used in schools. It is for this reason that I have developed m-Wizy<sup>TM</sup> cards to replace normal playing cards. These cards substitute a complete set of playing cards – the heart, diamond, spade and club suits featured in poker cards are replaced with square, triangle, rectangle and circle in m-WizyTM cards. In the same way, Jack, Queen and King are replaced with cards numbered 11, 12 and 13. There are also four blank cards that serve as 0 (zero) or Jokers when needed.

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# **Research news in MCG**

Demetra Pitta-Pantazi - Section Editor

# Exploring potential of professional learning communities of teachers to meet the needs of gifted learners

# Viktor Freiman, Jean Labelle, and Yves Doucet,

#### Université de Moncton, Canada

In this short article, we share our on-going research on the work of professional learning communities (PLCs) of teachers to support all learners including the gifted ones. At the first stage of our work (2011-2012), we conducted interviews with six classroom teachers from one French school district in New Brunswick, Canada, in order to collect their perceptions on the role and potential of PLCs as a mechanism of support of mathematically gifted students. We asked them three specific questions about (1) how gifted students in general, and specifically in mathematics, are identified by PLCs; (2) how PLCs help focus on strengths and challenges of these students; and (3) what measures are taken by PLCs to meet the needs of gifted students, especially in mathematics. In the report, we give a brief overview of results, still at the stage of analysis. This will be extended in a full paper that we are planning to present at the next annual meeting of the American Educational Research Association.

The particularity of the context of francophone minority schools in New Brunswick's duality (French and English) brings several challenges to the school system, such as geographic disparity (remote under-populated rural schools versus few over-populated urban ones) with schools often significantly distant from one another, demographic decline, lack of resources (human, material and financial), and cultural identity issues (Landry, Allard, & Deveau, 2010). According to Bajard (2009), this may negatively affect gifted learners who would have fewer chances to get recognized and appropriately nurtured than their peers living in language majority settings and with better socio-economic status. A school inclusion system, where all students are expected to attend regular classes and learn according to the same curriculum, leaves schools and teachers with additional challenges, in order to meet a variety of learning styles and abilities. (MacKay, 2005).

With the government strategy called Kids Come First (NBED, 2007), new opportunities were created for innovative teachers and gifted learners in the projects supported by Innovative Learning Funds. In some instances, we conducted innovative programs in schools where mathematically gifted students could be provided with opportunities to follow enrichment programs, to create problems for the virtual problem-solving community CAMI using multimedia tools (<u>www.umoncton.ca/cami</u>; Manuel and Freiman, 2012), to learn robotics (Blanchard et al., 2009), and to enjoy challenging mathematics in regular classrooms (Freiman, 2010). At one school, enrichment was followed by acceleration for several students who could skip one grade (moving directly from Grade 8 to Grade 10 mathematics). Within a recently completed follow-up study with these students and their teachers by Doucet (2012),



the issue of teachers' preparation to work with gifted (already raised in Applebaum, Freiman, and Leikin, 2011) resurfaced in the connection to the PLCs that are being established in all provincial schools.

According to Porter and Stone (1998), the formation of small teams is a way of achieving the inclusion of all students, even gifted ones. Similarly, Leclerc (2009) indicates that the PLCs would ensure that all students, without exception, get the services they need to succeed.

According to Hord (2004), « Professional Learning Community » is a recent concept in the literature promoting an idea of grouping professionals (in our case, teachers) who desire to acquire and share knowledge to further their understanding and better their practices. It is conceived as a mode of operation for schools that puts emphasis on collaboration of all actors and encourages all school professionals to work together towards reflecting and planning common actions to improve learning results of all students (Leclerc, et al., 2009). Such a school model has four major axes: learning instead of teaching, collaboration instead of competition, results instead of intentions, and evidence instead of opinions (Eaker, DuFour, & DuFour, 2004). Moreover, according to DuFour, DuFour, and Eaker (1998), PLCs should provide a structure to answer three fundamental questions: (1) what do we want students to learn, (2) how will we know students have learned, and (3) what do we do if students have not learned what their were supposed to learn or do know it before the teaching begins (already know). The third question refers specifically to the issue of meeting needs of diverse learners including the gifted ones.

The lack of research on linking giftedness to the PLCs, in general, and in the context of New Brunswick, in particular, prompted us to conduct a small case study in one school district of the province. While having a possibly wide range in structure and functioning of PLCs in different schools, even within one district, there is homogeneity in context (said district is the most affected by minority issues) and pedagogical approaches (through common professional development).

Ensuring representation of different school levels (K-12), we had two participants from each school type: elementary, middle and high school. Participants were invited by the authors based on their interest in sharing perceptions about gifted students. Six semi-structured interviews (30-40 minutes long) were conducted by one of the authors, audio-recorded and then transcribed by a student-assistant. A grid with some initial questions was used for the interview allowing for setting-up initial (general) questioning, as well as more specific (and open-ended) conversation around our research questions. A thematic analysis of the interview transcripts was done by one of the authors and validated by the third in order to ensure an internal validity of data interpretation. The analysis was conducted for each participant separately, and then was cross-analyzed as a way of triangulation among schools and school levels.

Following our preliminary data analysis, our findings were grouped around our three research questions, namely: how gifted students in general, and specifically in mathematics, are identified by PLCs; how PLCs help focus on strengths and challenges of these students; and what measures are taken by PLCs to meet the needs of gifted students, especially in mathematics.

First, according to our participants, identification of gifted students mainly happens within identification of students with special needs. Already in kindergarten, some students can be referred by their teachers to be assessed by the resource-teacher, and sometimes by a psychologist. As a result, a portfolio is created and follow-up measures pursued until the



end of schooling. Later, an individual intervention plan may be set-up by a strategic team composed of a member of the school's administration, the resource-teacher, the school psychologist, and the classroom teacher.

As such, PLCs do not seem to focus on students with special needs. Their main focus is rather on students 'at risk', meaning those who seem to have success potential but may fail if not supported on time. Those students are identified by means of diagnostic, formative and summative assessments. By looking at helping students 'at risk', teachers work in PLCs by setting SMART (specific, measurable, attainable, realistic and timely) objectives. During a school year (or semester), the PLCs go through several cycles of reflection and analysis (Figure 1, Labelle, 2010). At best, measures undertaken would serve not only students at risk, but all others as well.



#### Figure 1. Professional Learning Community Cycle in Education (Labelle, 2010).

Further, our participants mentioned that, since PLCs do not focus on students with special needs, identification of gifted students, if it happens, does so by accident as special cases of under-achievement or misbehavior. Similarly to the first question, answers to our second question seem to indicate that strengths and challenges of gifted students are not part of the PLCs' work, as they focus on a different group of students. Moreover, our participants mentioned that this would not be a subject of concern in strategic teams either.

Finally, the third question helped us learn that while several measures exist to help students with special needs presenting severe difficulties or handicaps (such as diagnostic, financing, human resources, material, including technology) within strategic teams and PLCs, little is done specifically for the gifted. Some schools do offer enrichment tasks, on which students work alone; others would use more structured activities (like real-life projects). None of these initiatives, however, are related to PLCs which mostly make room for informal discussions and sharing of experiences.

As preliminary conclusions, regarding teachers' perceptions of the PLCs and their role in identification of gifted learners, their needs and how to meet them, our data revealed that 'at risk' students - those who need immediate and appropriate assistance from teachers in order to achieve a passing grade - appear to be the main concern. This does not mean that teachers are indifferent to needs of the gifted students; however they refer to 'strategic teams' as a mechanism to support this category of learners with special needs. Although our data provided with a rather limited perspective on possible partnerships between special education professionals (as part of strategic teams) and classroom teachers (as part of PLCs)



to coordinate efforts of more inclusive teaching of learners with special needs, there are many positive signs of enthusiasm and commitment.

Also, in relation to particular needs of gifted students, teachers' perceptions seem to confirm general statements of shortage of appropriate resources and training, as well as time pressure. Teachers believe that PLCs may help them improve their inclusive practices through differentiation, enrichment, and challenge. They claim to be ready to look for and share new approaches and to conduct rigorous analysis of best practices, but still need better accompaniment and collaboration in order to implement them in their teaching, thus opening another door for further research.

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# "High IQ and High Mathematical Talent!"

# Results from nine years talent search in the PriMa-Project Hamburg

Summary of a report given at 12<sup>th</sup> International Congress on Mathematical Education

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## Introduction

Since school year 1999/2000, at the University of Hamburg we are fostering mathematically talented children of year three and four within the framework of the project called PriMa. This project is a research project and a project for fostering mathematically talented children.<sup>1</sup> Each year, we are fostering 50 children of grade three and 50 children of year four. At the moment (2012), in Hamburg there are around 12.000 children of year three.

To detect among them mathematically especially talented children, demands a highly comprehensive search for talents. Due to the age of the children, we decided to apply a multi-level procedure. As a first step trial lesson at one weekend offer the possibility to learn more about our kind of tasks. This is followed by a mathematics test. By that mathematics test, mathematical abilities are tested especially by focusing on the ability to recognise patterns and structures as well as the ability to generalise. After that, an intelligence test is executed (Nolte 2004). The results from the trial lessons, the mathematics test and the intelligence test are balanced against each other in order to compose groups. The 50 best performing children get a place in the university project while all other children get a place in one of the 60 mathematics workshops (*Mathezirkel – mathematics circle*) which are already implemented all over Hamburg at various primary schools for children who's interest in mathematics goes beyond the scope of ordinary mathematics lessons.

## Wouldn't an intelligence test be sufficient?

The idea to use mathematics tests and intelligence tests arose from the question, how far intelligence tests are applicable to predict mathematical abilities. Cognitive components such as the recognition of patterns and structures, inductive thinking, reasoning, processing of visual-three dimensional information are essential in mathematical problem solving.

However, these components are also described as components which are collected by intelligence tests (Sternberg 1986; Rost 2009). Nevertheless, it must be asked "do intelligence tests really cover the complexity and kind of mathematical thinking that is needed to solve our kind of problems?" (see for example Dörner (1992); Nolte 2004; Putz-Osterloh / Lüer 1981;Waldmann and Weinert 1990).

<sup>&</sup>lt;sup>1</sup> PriMa is a cooperation project of the *Hamburger Behörde für Schule und Berufsbildung* (u.a. *BbB*), and the *William-Stern* Society (Hamburg), the University of Hamburg. (for further information you are invited to visit the website http://blogs.epb.uni-hamburg.de/nolte/)



## Is an intelligence test suitable to detect special mathematical talent?

During the last 13 years, more than 2000 children run though our talent search. During these years we observed that from a high IQ we could not automatically predict high mathematical performance in our groups. For the beginning, these described observations were mainly based on single cases. Therefore a number of single cases were scrutinized by Björn Pamperien: the results of the intelligence tests of the best mathematically performing children were listed, and, the other way round, he listed the results of the mathematics performance of the children with the best results in the intelligence test.



Figure 1. Results of two boys: IQ and the number of Points in the Mathematics Test (MTP) (Year 1)

We do not know the reasons for such a low level of performance of the first boy. The second boy performed as one of the best in the mathematics test. We also got inverse results. In the majority of cases a high IQ comes along with high mathematical abilities. But the single cases show that from a high IQ we shall not automatically conclude a high mathematical performance or the in the opposite way, from good mathematical performance one cannot conclude a high IQ.

These results induced us to calculate the correlations of the results of the mathematics tests and those from the IQ tests for being able to get more general answers about the single cases. The calculations and the first interpretations were made by (Pfennig 2009).

## Method

In the first years, besides the mathematics test, we used the KFT 1-3 and CFT 20 for testing the children. During the last years, we only used the CFT 20 R in connection with a vocabulary test and a number sequence test. The CFT 20 (R), which is a language-free test, is suitable for testing children whose mother tongue is not German. This test maps the general level of intelligence (factor "g")(Weiß 2006). The first year we did not take into further consideration because that year we just got our first experience with our described procedure. Thus, we took the results of the second up to the tenth year (G2 until G10). And only those children could be taken into consideration from whom we had complete results of both tests, meaning children who participated in the mathematics test and in the intelligence



test as well. Likewise, we could not use results from children who were too young. The CFT 20 is standardised for children from the age of 8,5 years. For 8,0 to 8,4 year old children the IQ had to be noted as approximate value. These children were not included into the evaluation. We only considered the results of CFT 20, because we applied the KFT 1-3 only in the first years. In total, the data of 1.663 children were calculated.

In order to make the results of the mathematics test comparable to the results of the intelligence test, we set up a ranking list of mathematics test points. The same total number of points was assigned to the same rank. A lower rank complies with very good performance. This kind of scaling enables a comparison of results from several years without depending on the highest number of points which has been attained, and takes into account the fact that in the course of time we refined our evaluation criteria of the mathematics test.

## Results

As there are no significant differences between the results of the years, for further evaluation only the whole group was taken into account.

Correlated variable	Correlation	Confidence- intervall	Squared correlation	Partial correlation
Rank mathematics test - CFT	-0.34	-0.30 to -0.39	11.8%	-0.28
Rank mathematics test – number sequence	-0.43	-0.37 to -0.48	18.2%	
Rank mathematics test - vocabulary	-0.24	-0.18 to -0.30	5.8%	

Figure 2. Correlations mathematics test, number sequences test and vocabulary test - complete sample

Correlated variables	Correlation	Confidence- intervall	Squared correlation	Partial correlation
Rank math test - CFT	-0.02	0.08 to -0.13	0.1%	-0.01
Rank math test - number sequences	-0.15	-0.02 to -0.29	2.3%	
Rank math test - vocabulary	0.03	0.19 to -0.12	0,1%	

Figure 3. Correlations mathematics test, number sequences test and vocabulary test - selected children



The correlation between CFT and the mathematics test is medium, which indicates that both tests measure intelligence. The squared correlation states that only 11.8 % of the deviations from the mean value in the mathematics test through deviation in the IQ-Test (and the opposite way) can be explained. It means that 11.8 % of the results of the mathematics tests can be predicted by means of the CFT 20. The correlation between number sequences and the mathematics test is significantly higher, certainly due to the fact that number patterns had to be recognized in the mathematics test.

The calculation of the partial correlation is aimed at indicating the correlation between the mathematics test and the CFT, if that part of the correlation caused by the correlation through testing "number sequences" in both tests is taken out. The fact that the correlation is slightly lower suggests that by the mathematics further aspects besides number sequences are tested.

With the selected children, the correlation between mathematics test and IQ test is significantly lower. This is because for the participation in the project results from the *Mathe-Treff für Mathe-Fans* - *Mathematics Meeting for Mathematics Fans* were included. This discrepancy occurs because the 20 best performing children of the mathematics test were accepted independently from their results of the IQ test.

## Interpretation

Because of the described results, we assume that mathematical talent can be measured by our mathematics test. Therefore, the following statements must be regarded from this point of view. First, it must be stated that both tests correlate with each other from medium up to strongly. Although the math test is more complex than a "normal" IQ test and some specific aspects of mathematical abilities are recorded by it, both tests compile data about the construct "intelligence". Therefore, there exists a strong correlation between both tests, but which due to the specifics of the mathematics test only a medium up to strong correlation is indicated. However, as the correlation is not higher means that different aspects are measured by the tests.

If one calculates the correlation between the tests for the children which are included in the 25%, the correlation is significantly lower. If a rank of 15 or better "is taken as reference (standard) for mathematical talent " the correlation is 0.14" (Pfennig 2009). The correlation between the tests is much weaker and very often not significant with these selected children. On the one hand, this is because of the small sample, and, on the other hand, motivational factors were also considered for the selection of the children. Children, who made clear for us that they only participated in the procedure, because their parents wanted them to come (parents' will), were not listed for the project.

## Does giftedness always come along with mathematical ability?

Generally, children classified as being talented by the CFT test perform better in the mathematics test than other children (not talented according to the CFT test). CFT-gifted children on average are achieving a rank of 37 in the mathematics test while children not classified as gifted achieve on average a rank of 50. These differences are statistically not coincidently. However, the reduction of correlation with a broader selection or a homogenisation of the group indicates that what we record as mathematical talent, cannot be deduced from the IQ. Therefore, the question whether talent always goes hand in hand with a high mathematical talent, can clearly be answered with "no".



However, the reduction of the correlation has to be expected. The less selected the measured population, the more exact are the intelligence tests. Our results may be caused by the fact that general intelligence (g) in our group is not well differentiated. "In unselected populations, variance of g is always stronger than all other additional excluded factors taken together". (Rost 2009, p. 48). Children come to us whose IQ is between 66 and 160, but no normal distribution exists. This kind of selection causes a shift of the normal distribution to which intelligence tests refer to.

IQ	Persons i unselected group	n	Persons in the PriMa-Project	Percentage rank PR ca.
70 -79	ca. 7 %	読ん	ca. ½ %	2-8
80-89	ca. 16%		ca. 2%	9-23
90-109	ca. 50%		21%	25-73
110-119	ca.16%		25%	75-90
120-129	ca.7%	1	24%	91-97
130 and more	ca. 2%		27,5%	98 and more

Figure 4: Table (Data according to Rost 2009, S. 153)

The majority of the children come to us, because they are interested in mathematics and / or parents or teachers think that they are mathematically talented. A further selection is that children first participate in the Mathematics Meeting for Math Fans where they find out whether they are willed or able to work on complex mathematical problems for a time of 90 minutes which is quite unusual for this age. Children apply for the test who enjoyed participating and feel that they are qualified enough for such a test, which means they feel that they perform better in mathematics than many other children. This is not always the case for girls. If a friend quits the talent search process even girls with high mathematical abilities tend towards quitting themselves. Nevertheless the group of tested children is a highly selected group.

How tall basketball players are is used by Rost (2009) to illustrate the problem that may occur through homogenisation of a group in connection with statements about correlation. In an unselected sample how tall a person is correlates positively with the ability to pitch a ball into the basket. The taller a person, the more hits of baskets can be expected in a basketball game. However, if one considers basketball players who are playing as professionals how tall they are do not show anything, because all professional players are very tall.

Compared with our group this means that from a certain degree of mathematical giftedness the IQ does not differentiate any more concerning mathematical capacity. On the one hand, this proves the experiences of mathematics didactics experts who work with mathematically talented children: An intelligence test does not record differentiated enough whether very high performance in mathematical problem solving processes may be presumed for these children.



This does not contradict with what has been found out by psychologists: that intelligence tests executed in an unselected sample identify with a great degree of accuracy children with special mathematical talent also as highly intelligent.

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# How to contribute

All MCG members (and non-members) are welcome to contribute short articles to the MCG newsletter. The authors are invited to submit articles (from 500 to 3000 words) to a Section Editor (see below) and the chief editor of the newsletter – Roza Leikin [*rozal@edu.haifa.acil*].

Each contribution will be reviewed by the head of the section, one of the board members and one additional expert in the field. The newsletter is published in electronic form and is distributed electronically by e-mail and placed on a special page of MCG web site. **The newsletter includes five sections:** 

#### **Educational innovations and history**

#### Section Editor: Alexander Karp [apk16@columbia.edu]

This section of our Newsletter is intended to help all of us keep abreast of pedagogical and organizational innovations in the education of the mathematically gifted and in the development of creativity, as well as about the history of such education and development

#### Mathematical competitions corner

#### Section Editor: Peter Taylor [pjt013@gmail.com]

In this section, we plan to publish ideas about mathematical competitions and their role in mathematics education. We wonder what role mathematical competitions play in the advancement of mathematical creativity and giftedness and how expertise and creativity are combined when solving Olympiad problems. We also plan to discuss different forms of mathematics competitions.

#### **Problem corner**

#### Section Editor: Romualdas Kasuba[romualdas.kasuba@maf.vu.lt]

We welcome authors to publish their original problems as well as original solutions to known problems. We are interested in the discussion of problems that raise students' motivation in learning mathematics as well as such characteristics as beauty and elegancy of the solutions.

#### **Creative news in mathematics**

#### Section Editor: Emilia Velikova [evelikova@uni-ruse.bg]

This section will include information about mathematical and technological innovations concerning Creativity in Mathematics Education. We are welcome papers about history of mathematical creativity, new discoveries in mathematics, methods for creating new problems and original solutions and the use of new technologies in mathematical inventions.

#### **Research news in MCG**

#### Section Editor: Demetra Pitta-Pantazi [dpitta@ucy.ac.cy]

This section of Newsletter presents information on research in the field of mathematical creativity and giftedness including a digest of an on-going research, initial findings of unpublished research, or reviews of published research papers and books that focus on empirical and theoretical issues of mathematical creativity and giftedness. This section also can be used for announcements of proposals for research collaboration by those who search for research teammates.



# *Information page* International Group for Mathematical Creativity and Giftedness

The purpose of the group is to bring together mathematics educators, mathematicians, researchers, and others who are interested in nurturing and supporting the development of mathematical creativity and the realization of mathematical promise and mathematical giftedness, promoting the improvement of teaching and learning mathematics, and widening students' abilities to apply mathematical knowledge in innovative and creative ways.

The work of the Group will concentrate on distinct but interrelated **domains** such as:

- mathematical creativity for all students, from all backgrounds, and of all ages,
- mathematical creativity, aptitude, and achievement,
- mathematical giftedness, talent and promise,
- mathematical creativity for individuals or teams, inside or outside the classroom,
- mathematics competitions,

#### The aims of the group are to:

- encourage research concerning the discovery, nurture and support of mathematical creativity, giftedness, talent and promise for all students,
- support investigation and dissemination of information on the role of teacher knowledge and education, educational systems, and cultural aspects related to the development of mathematical creativity and promise,
- stimulate national and international activities to further the aims of the Group,
- cooperate with national and international associations with similar aims,
- organize international conferences and stimulate discourse between mathematicians, psychologists, educators, researchers, curricular designers and sociologists,
- stimulate and support members of the Group to participate actively at conferences or projects or other activities that further the aims of the Group
- further scientific publications and encourage the development of websites,
- assist educators and inform policy-makers,

**MCG is affiliated with <u>ICMI</u>** (the International Commission on Mathematical Instruction), see details at: <u>http://www.mathunion.org/icmi/about-icmi/affiliate-organizations/study-groups/mcg/</u>

More information about the Group can be found at MCG Website <a href="http://igmcg.org/">http://igmcg.org/</a>

To join the group complete membership form at MCG website

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